Mécanique statistique classique — 30 septembre 2009

Série 2

Problem 1

a) Starting from the formula for the microcanonical entropy of the ideal gas

$$S(E,V,N) = Nk_B \left\{ \frac{3}{2} + \ln \left[V \left(\frac{4\pi m E}{3Nh^2} \right)^{3/2} \right] \right\},$$

derive the internal energy $E(S,V,N)$.

b) Explain the mathematical meaning of the expression

$$\mathrm{d}E = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N$$

and show that

$$p := T \left(\frac{\partial S}{\partial V} \right)_{E,N}, \quad \mu := -T \left(\frac{\partial S}{\partial N} \right)_{E,V}.$$

c) Using the formula

$$p = -\left(\frac{\partial E}{\partial V} \right)_{S,N}$$

show that for the ideal gas, the equation $pV = NkT$ holds.

Problem 2

Find the microcanonical entropy $S(E,V,N)$ for a system consisting of N distinguishable classical harmonic oscillators with identical frequencies ω. Compute the heat capacity

$$C := \left(\frac{\partial E}{\partial T} \right)_{V,N}$$

of the system.

Problem 3 (optional)

According to Eq. (30) in the lecture notes, volume of the N-dimensional sphere with radius R is

$$V_N = \frac{\pi^{N/2}}{(N/2)!} R^N.$$

a) If this formula was not derived during the lecture, try to derive it. (The derivation is tricky—it's better to directly consult the lecture notes, page 9, or Greiner's book, example 5.2 on page 129 of the English edition.)

b) Given an N-dimensional sphere with radius R, find what fraction $x(N)$ of its volume is contained in the outer shell with thickness $R/100$.

c) Draw $x(N)$ as a function of N. What is the limit $\lim_{N\to\infty} x(N)$?